REPRESENTATION THEORY MID TERM

This exam is of **30 marks** and is **3 hours long**. Please **read all the questions carefully**. Please feel free to use whatever theorems you have learned in class after stating them clearly. Please sign the following statement:

I have not used any unfair or illegal means to answer any of the questions in this exam.

Name: **Signature:**

1. Let G be a group and V_1 and V_2 two representations. Define a representation of $G \times G$ on $V_1 \otimes V_2$ as follows:

$$(g_1, g_2) \cdot (v_1 \otimes v_2) = g_1 \cdot v_1 \otimes g_2 \cdot v_2$$

This is called the *external tensor product* and is denoted by $V_1 \boxtimes V_2$.

(1) Show that if V_1 and V_2 are irreducible so is $V_1 \boxtimes V_2$. 5

(2) What is the character of $V_1 \boxtimes V_2$? Justify your answer. 5

(3) Let $\Delta(S_3)$ denote the diagonal subgroup of $S_3 \times S_3$. What is

$$Res^{S_3 \times S_3}_{\Delta(S_3)}(V_3 \boxtimes V_3)$$

where V_3 is the standard representation? 5

2. Express the character of $Sym^2(V)$ where V is a representation of G in terms of the character χ_V. 5

3. Decompose $Sym^2(V_4)$ in to its irreducible components, where V_4 is the standard representation of S_4. 5

4. Show that all representations of S_d are real. 5

Character Tables

	1	3	2
$\mathfrak{S}_3$	1	(12)	(123)
trivial U	1	1	1
alternating U'	1	−1	1
standard V	2	0	−1

	1	6	8	6	3
$\mathfrak{S}_4$	1	(12)	(123)	(1234)	(12)(34)
trivial U	1	1	1	1	1
alternating U'	1	−1	1	−1	1
standard V	3	1	0	−1	−1
$V' = V \otimes U'$	3	−1	0	1	−1
Another W	2	0	−1	0	2